

foundation not withstanding, the effects themselves may seem a test of plausibility. Length contraction and time dilation may seem to be a kind of descriptive slight of hand in which, simply by reorienting our perspective on nature, things *seem* to shrink and time *seems* to slow down. The symmetry of the effects in particular is unnerving. I say your time goes slow, and you say my time goes slow, and we are both right. This seems to invite the sort of contradiction that would show that we are not talking about the way things really are. At best we would be describing how things appear to different people.

The exposition of alleged or apparent contradictions in the special theory of relativity is usually done under the heading of a paradox. There is the famous Twin Paradox, the nearly as famous Pole-in-the-Barn Paradox, and others. These are all paradoxes in the sense that the situations they describe *seem* to entail contradictions, but on careful analysis there is no contradiction after all. They are all valuable cases to study because they challenge and sharpen the understanding of the special theory of relativity, and they show how its description of nature is entirely consistent.

One of the best paradoxes is not famous at all. It was suggested by my friend Mitch, so I call it Mitch's Paradox. It is inspired, as is the Twin Paradox, by the perplexing symmetry of time dilation. Mitch's Paradox involves two people born at the same time, but in significantly different circumstances than the Twin Paradox. The more famous case has the distinct disadvantage of involving a non-inertial reference frame and thereby going outside the domain of the special theory of relativity. Everything is inertial in Mitch's Paradox, and we can solve the initial puzzle quite easily.

There are two men, Bob and Bubba, who are exactly the same age and who live in neighboring towns along the railway line. They remain at rest in reference frame K, separated by a distance  $d$  (as measured in K). Another person, Richard, passes by, moving along the line separating Bob and Bubba. When Richard passes next to Bob, we note that Richard and Bob are exactly the same age. The challenging question is this: When Richard then passes by Bubba, will Richard be younger than Bubba, older than Bubba, or the same age as Bubba? The challenge is generated by the symmetry of time dilation. On the one hand, from the reference frame K of Bob and Bubba, Richard will age more slowly, so Richard will be younger than Bubba. But on the other hand, from Richard's own reference frame K', Bob and Bubba should age more slowly and Richard will be older than Bubba. Either one of these arguments is wrong (and we have to figure out which one), or the special theory of relativity contains a contradiction (and it cannot describe how nature really is). Richard cannot be both younger and older than Bubba. There is an absolute fact of the matter about their ages at the moment they are together. We can take a photograph of the two of them together as Richard speeds by, and ei-

ther Richard or Bubba will have more gray hair and more wrinkles. This photograph will show unambiguously who is older, and it can be passed from one reference frame to another without alteration. The two frames must agree on who is younger at the event of Richard and Bubba being together.

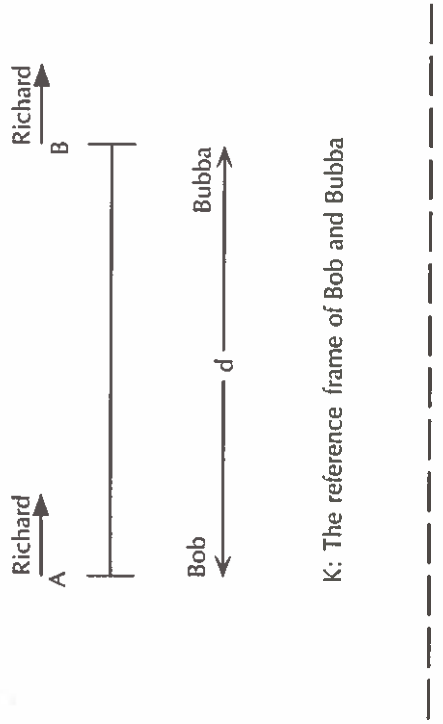
The answer is that Richard is unambiguously younger than Bubba when the two are at the same point. Bubba will say that Richard looks younger than Bubba himself, and Richard will say that Bubba looks *older* than Richard himself.

This result does not violate the symmetry of time dilation because the procedures for measuring the time duration between the two events are not the same for the K frame and the K' frame. The two events in question are the meeting of Richard and Bob (event A) and the meeting of Richard and Bubba (event B). In Richard's reference frame K', the two events happen at the same place, namely at Richard's position. In the reference frame of Bob and Bubba, that is, in the K frame, the two events happen at different places, one at Bob's position and the other at Bubba's. The symmetry in the time dilation effect referred to a situation in which two different frames did *identical* time measurements on each other. Because of the difference in this case between the kinds of measurements done in the two frames, we should not expect symmetry.

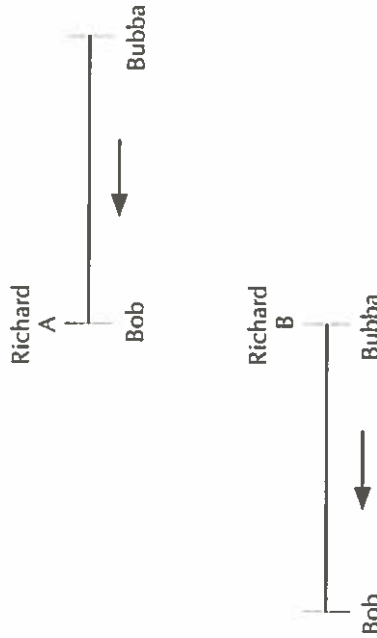
The resolution of Mitch's Paradox can be explained either with a Minkowski diagram or a look at the physical situation. Figure 3.12 is a stylized picture of the physical situation, both in the K frame and the K' frame. The key to understanding why Richard is younger than Bubba when they meet is remembering length contraction. The distance between Bob and Bubba is  $d$  in the K frame, but it is shorter than  $d$  in the K' frame. As Richard moves by, the distance between Bob and Bubba contracts. So Bob and Bubba see Richard move at some speed for a length  $d$ . Richard sees Bob and Bubba moving at the same speed but the length between them is less than  $d$ . It takes less time for the Bob-Bubba segment to pass Richard than it takes Richard to move from Bob to Bubba. Thus, the duration of time between event A and event B is less for Richard than it is for Bob and Bubba. Richard ages less between these events. Richard is younger than Bubba.

But if everyone agrees that Richard and Bob are the same age at event A, and that Richard is younger than Bubba at event B, then, since Bob and Bubba are the same age, doesn't everyone agree that Richard is aging more slowly than Bob and Bubba? That is, isn't Richard's time absolutely slower? No, and the Minkowski diagram is useful for understanding why.

To say that Bob and Bubba are the same age is to say that they were born at the same time. This is an issue of simultaneity and so it is relative. The K' observer will not agree that Bob and Bubba were born at the



K: The reference frame of Bob and Bubba



K': The reference frame of Richard

Figure 3.12

same time since the events of their birth, though simultaneous in the K frame, are not simultaneous in the K' frame. As shown in Figure 3.13, using K' lines of simultaneity, Bubba was born before Bob, and Bubba is always older than Bob. That is why Bubba is older than Richard when those two get together. Bubba was older than Bob when Richard passed Bob.

The moral of this story is that the special theory of relativity does not lead us into any contradiction of the form that Richard is both younger and older than Bubba. When two people get together, however briefly, there will be a biological fact of the matter as to how much aging their bodies have done. Relativity does not ask us to ignore these biological,

or other physical facts. If Bubba and Richard were piles of radioactive atoms instead of piles of organic molecules, we could note exactly how many in each pile had decayed at the moment of their being together. More of Bubba would have decayed, period.

### NOTHING CAN GO FASTER THAN THE SPEED OF LIGHT

The speed of light is absolute, invariant in all inertial reference frames. The speed of light is also an upper limit on the speed of any causally effective physical signal. No object can go faster than the speed of light. No information in any form can go faster than the speed of light.

This second aspect of the speed of light, that it is a universal upper limit, is often cited as another postulate of the special theory of relativity. But we do not have to accept it as a postulate; it can be proven. Again, the Minkowski diagram is the key to clarity on this issue.

As a concrete example to work with, consider the speed of a baseball that is tossed toward a window. Say that event A is the event of the ball being tossed, and event B is the event of the ball hitting and breaking the

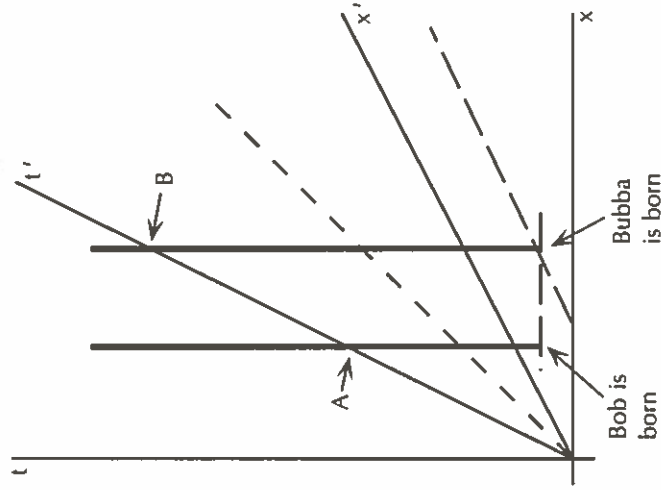


Figure 3.13