1) (10 pts) Work-energy principle/thereom:

a) State the work-energy principle with an equation.
\[ W_{nc} = \Delta KE + \Delta PE \]

b) Explain (in words) the meaning of the principle.
Net work done by non-conservative forces on a system is equal to the system's change in kinetic energy plus change in potential energy.

c) What can you say about a physical system that experiences only conservative forces?
\[ W_{nc} = 0 \quad \text{so} \quad \Delta KE = -\Delta PE \]
Total mechanical energy is conserved!

d) Give an example of such a system. Describe briefly in words, and use a diagram if you like also.
System: object falling to earth without friction.
Converts initial \( PE = mgh \) to \( KE = \frac{1}{2}mv^2 \) before striking ground. Gravity is a conservative force, so \( W_{nc} = 0 \)

2) (10 pts) Three different balls of masses 10 kg, 20 kg, and 30 kg are launched from the roof of a building (height = 100 m) at angles of 10°, 20°, and 30°, respectively, to the horizontal. All three balls are launched with an initial speed of 10 m/s. Which ball strikes the earth below with the greatest speed? Explain/show your work.

\[ v_0 = 10 \text{ m/s} \]
Conservation of total mech. energy
All 3 balls have \( E = mgh + \frac{1}{2}mv^2 \)
So \( mgh + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 \)
\( v^2 \) independent of mass!

All 3 strike ground at same speed!
3) (10 pts) A 220-kg load is lifted 21.0 m vertically with an acceleration $a = 0.150 \text{g}$ by a single cable.

a) Find the tension in the cable.

\[ T = m \left[ 0.15 \cdot g + g \right] = m \cdot 1.15 \cdot g \]
\[ = (220 \text{ kg}) (9.8 \text{ m/s}^2)(1.15) \]
\[ = 2479 \text{ N} \]
\[ = 2.5 \cdot 10^3 \text{ N} \]

b) Find the net work done on the load.

**Assume a = constant**

**net Work = net force \cdot distance = m \cdot a \cdot d**

\[ = (220 \text{ kg}) (0.15) \cdot 21 \text{ m} = 6791 \text{ J} \]
\[ \sim 6.8 \cdot 10^3 \text{ J} \]

c) Find the work done by the cable on the load.

\[ W_{\text{cable}} = F_{\text{cable}} \cdot d = T \cdot d = (2479 \text{ N})(21 \text{ m}) = 52,059 \text{ J} \]
\[ = 5.2 \cdot 10^4 \text{ J} \]

d) Find the work done by gravity on the load.

\[ W_{\text{gravity}} = mg \cdot d \cdot \cos(180^\circ) = -mgd \]
\[ = -(220 \text{ kg}) \cdot 9.8 \cdot 21 \text{ m} = -45,270 \text{ J} \]
\[ = -4.5 \cdot 10^4 \text{ J} \]

e) Find the final speed of the load assuming it started from rest.

\[ v^2 = v_0^2 + 2a \cdot d \]
\[ v_0 = 0 \]
\[ v^2 = 2 \cdot 0.15 \cdot (9.8 \text{ m/s}^2)(21 \text{ m}) \]
\[ v = 7.9 \text{ m/s} \]

(can also solve using energy conservation!)

4) (10 pts) Is it possible for a body to receive a larger impulse from a small force than from a large force? Explain, briefly, using Newton's 2nd law.

**Newton's 2nd Law**

\[ F = \frac{\Delta p}{\Delta t} \]

impulse = \Delta p = Fat

Yes - a body can receive a larger impulse from a small force than from a large force if the small force acts on the body for a longer time \(\Delta t\)! So \( F_{\text{small}} \cdot \Delta t_{\text{long}} \) can be

\[ > F_{\text{large}} \cdot \Delta t_{\text{small}} \]
5) (10 pts) An 18-g projectile traveling 230 m/s buries itself in a 3.6-kg pendulum hanging on a 2.8-m long string, which makes the pendulum swing upward in an arc. Prior to the collision, the pendulum is hanging at rest at its lowest point.

   a) What kind of collision is involved?

   **Inelastic collision** - ballistic pendulum - see Ex 7.10 in text

   b) Determine how high the pendulum swings after the collision. Ignore friction.

   During collision, momentum conserved: \[ m_1 v_i = (m_1 + m_2) v' \]

   \[(0.018 \text{ kg})(230 \text{ m/s}) = (3.6 + 0.018) v' \quad v' = 1.144 \text{ m/s} \]

   After collision, mechanical energy is conserved:

   \[ \frac{1}{2} (m_1 + m_2) v_i^2 = (m_1 + m_2) g h \quad \text{solve for} \quad h = \]

   \[ h = \frac{(1.144 \text{ m/s})^2}{2 \cdot 9.8} = 0.067 \text{ m} \quad \text{or} \quad (6.7 \text{ cm}) \]

6) (10 pts) An explosion breaks an object, originally at rest, into two fragments. One fragment acquires twice the kinetic energy of the other. Find the ratio of their masses. Show your work!

   Before explosion, \( p = 0 \). After explosion, \( p = 0 \), \( m_1 v_i = m_2 v_z \)

   But \( \frac{1}{2} m_1 v_i^2 = 2 \left( \frac{1}{2} m_2 v_z^2 \right) \) So \( m_1 v_i^2 = 2 m_2 v_z^2 \) Then \( \Rightarrow \)

   \[ \frac{m_1}{m_2} = 2 \frac{v_z^2}{v_i^2} \quad \text{From momentum conservation,} \quad \frac{v_z}{v_i} = \frac{m_1}{m_2} \quad \text{So} \Rightarrow \]

   \[ \frac{m_1}{m_2} = 2 \frac{m_1}{m_2} \quad \text{So} \quad \frac{m_2}{m_1} = 2 \quad \text{One that gains the larger KE is half as massive as other!} \]

7) (10 pts) A light body and a heavy body have the same kinetic energy. Which has the greater momentum? Show your work.

   Let \( m_1 < m_2 \) \( \frac{1}{2} m_1 v_i^2 = \frac{1}{2} m_2 v_z^2 \) Compare their momenta \( m_1 v_i = \frac{p_i}{m_2 v_z} \)

   \[ \frac{p_1}{p_2} = \frac{m_1}{m_2} \cdot \frac{v_i}{v_z} \quad \text{But} \quad \frac{v_i^2}{v_z^2} = \frac{m_2}{m_1} \quad \text{so} \quad \frac{v_i}{v_z} = \sqrt{\frac{m_2}{m_1}} \]

   So \( \frac{p_1}{p_2} = \frac{m_1}{m_2} \cdot \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}} < 1 \) So \( p_1 < p_2 \)

   Thus heavier body has the greater momentum!
8) (10 pts) A sphere and a cylinder of equal radius and mass start from rest at the top of an inclined plane. The moments of inertia are \( I_{cylinder} = \frac{1}{2} MR^2 \) and \( I_{sphere} = \frac{2}{5} MR^2 \).

a) Which reaches the bottom first? Explain, briefly.

\[ I_{sphere} < I_{cylinder} \]
\[ Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} Iw^2 \]

One with larger \( I \) will have smaller \( v \).

So sphere reaches bottom first!

b) Which has the greater total kinetic energy at the bottom? Explain, briefly.

Both have same total KE = Mgh + \( \frac{1}{2} Iw^2 \) since they both start with same total mechanical energy.

c) Which has the greater rotational kinetic energy at the bottom? Explain, briefly.

The cylinder. It has larger \( I \), so more of its total energy will be converted to rotational energy.

9) (10 pts) A small rubber wheel is used to drive a large pottery wheel, mounted so that their circular edges touch. The small wheel has a radius of 2.0 cm and accelerates at a rate of 7.2 rad/s\(^2\). It is in contact without slipping with the pottery wheel, whose radius is 25.0 cm. Find the time needed for the pottery wheel to reach its required speed of 65 rpm. Assume the pottery wheel starts from rest.

Find angular acceleration of pottery wheel. No slipping,

So the two linear accelerations must be equal \( \alpha = \alpha_r \)

\[ \alpha_r R = \alpha R \]

or \( \alpha R = \frac{(7.2 \text{ rad/s}^2)(2.0 \text{ cm})}{25 \text{ cm}} = 0.576 \text{ rad/s}^2 \)

Then: \( \omega_{\text{final}} = 2\pi(65 \text{ rpm})/(60 \text{ s/min}) = 6.8 \text{ rad/s} \)

\[ 6.8 \text{ rad/s} = \omega_0 + \alpha t \]

\[ = 0 + \left(0.576 \frac{\text{rad}}{\text{s}^2}\right)(t) \]

\[ t = 11.8 \text{ sec} \]
10) (10 pts) A hollow cylinder (hoop) is rolling on a horizontal surface at speed $v = 3.3 \text{ m/s}$ when it reaches a $15^\circ$ incline. How long will it be on the incline before it arrives back at the bottom?

Assume zero friction — conservation of energy:

$$v = \omega r$$

$$E_{\text{bottom}} = E_{\text{top}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mgh$$

or $h = \frac{v^2}{g} = \frac{(3.3 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 1.11 \text{ m}$

Distance along ramp $d = \frac{h}{\sin 15^\circ} = 4.3 \text{ m} = \Delta x$

Constant acceleration so $d = \Delta x = \frac{1}{2} (v + v_0) t$

$$t = \frac{2 \Delta x}{v + v_0} = \frac{2 (4.3 \text{ m})}{0 + 3.3 \text{ m/s}} = 2.6 \text{ s}$$

Time to go up $= 2.6 \text{ s}$

Total time $= (5.25)$

Useful Equations & Information

For $ax^2 + bx + c = 0$, solutions to the quadratic are $x = (1/2a) [-b \pm (b^2 - 4ac)^{1/2}]$

$v = v_0^2 + 2a(x-x_0)$

$x(t) = x_0 + v_0t + \frac{1}{2}at^2$

$v = v_0 + at$

$F_{\text{net}} = ma$

$G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$

$a_c = v^2/r$

$F = Gm_1m_2/r^2$

$W = F \cdot d = F \cdot d \cdot \cos \theta$

$F = \Delta p/\Delta t$

$p = mv$

$v = r \omega$

$a_{\text{tan}} = r \alpha$

$$\tau = I \alpha$$

$KE = \frac{1}{2} I \omega^2$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2 \alpha \theta$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$