For these problems use the same procedure you use for homework problems: describe the problem (including pictures, knowns/unknowns, principles and concepts), plan a solution, and carry out the plan. Partial credit will be given! Total possible points = 120.

1) (10 pts) A falling stone takes 0.3 seconds to travel past a window 2.2 m tall.

a) Find the velocity of the stone at the top of the window.

\[ d = V_0 t + \frac{1}{2} at^2 \]

\[ 2.2m = V_0 (0.3s) + \frac{1}{2} (9.8 m/s^2) (0.3s)^2 \]

Solve for \( V_0 = \frac{5.86 m/s}{\text{downward}} \)

b) Find the height above the top of the window from which the stone fell.

\[ V_0^2 = V_{\text{release}}^2 + 2a(dy) \]

\[ (5.86 m/s)^2 = 0 + 2(9.8 m/s) dy \]

\[ dy = \frac{5.86^2}{2 \cdot 9.8} = 1.75 m \sim 1.8 m \text{ above window} \]

2) (10 pts) The acceleration due to gravity on the moon is about 1/6 what it is on Earth. If an object is thrown vertically upward on the moon, how many times higher will it go than it would on Earth, assuming the same initial velocity? Explain, briefly.

\[ V_f^2 = V_0^2 + 2a(y - y_0) \]

Both cases \( V_f = 0 \), \( y_0 = 0 \)

\[ y = \text{max height} = \frac{V_0^2 - V_f^2}{2a} = \frac{-V_0^2}{2a} \]

Earth \( a = -9.8 m/s^2 \)

Moon \( a = -\frac{9.8}{6} = -1.6 m/s^2 \)

So \( y_{\text{max}} \sim 1/a \)

So on moon, where \( a_{\text{moon}} = \frac{1}{6} a_{\text{Earth}} \), \( y_{\text{max}} = 6 \text{ times higher} \)
3) (10 pts) Vectors:

a) Can two vectors of unequal magnitude ever add up to give the zero vector? If so, give an example.

No – if unequal, they can never cancel!

b) Can three vectors of unequal magnitude ever add up to give the zero vector? If so, give an example.

\[ \vec{A} + \vec{B} + \vec{C} = 0 \]

4) (10 pts) A projectile is launched with an initial speed of 75.2 m/s at an angle of 34.5° above the horizontal on a long, flat field.

a) Find the maximum height reached by the projectile.

At top, \( v_y = 0 \)

\[ v_y^2 = v_{y_0}^2 + 2a_y (y - y_0) \]

\[ 0 = (75.2 \text{ m/s} \sin 34.5°)^2 + 2(-9.8 \text{ m/s}^2)(h-0) \]

\[ h = \text{max height} = 92.6 \text{ m} \]

b) Find the total time spent in the air.

Total = 2 x time to fall 92.6 m

Time to go up:

\[ v_f = 0 = v_{y_0} - at \]

\[ t = \frac{v_{y_0}}{a_y} = \frac{(75.2 \text{ m/s})(\sin 34.5°)}{9.8 \text{ m/s}^2} = 4.35 \text{ s} \]

Total = 2 x 4.35 s = 8.7 s

c) Find the total horizontal distance traveled.

\[ \text{Range} = v_{x_0} \times \text{air time} \]

\[ = (75.2 \text{ m/s} \cos 34.5°)(8.7 \text{ s}) = 539 \text{ m} \]
5) (10 pts) An automobile traveling 95 km/hour overtakes a 1.0-km-long train traveling in the same direction on a track parallel to the road.

- \( V_{\text{car}} = 95 \text{ km/hr} \)
- \( V_{\text{train}} = 75 \text{ km/hr} \)
- \( t = 0 \)

\[ \begin{align*}
  x_{\text{car}} &= v_{\text{car}} \cdot t \quad \text{(front of car)} \\
  x_{\text{train}} &= 10^3 \text{ m} + v_{\text{train}} \cdot t \quad \text{(front of train)} \\
  v_{\text{car}} \cdot t &= 1000 + v_{\text{train}} \cdot t \\
  t &= \frac{1000}{v_{\text{car}} - v_{\text{train}}} = \frac{1000 \text{ m}}{20 \text{ km/hr}} = \frac{1 \text{ km}}{20 \text{ km/hr}} = \frac{1}{20} \text{ hr} = 3 \text{ min} \\
  t &= 3 \text{ min} \text{ or } 180 \text{ sec}
\end{align*} \]

a) If the train’s speed is 75 km/hour, how long does it take the car to pass the train?

b) How far will the car have traveled in this time?

\[
\text{distance} = \text{rate} \cdot \text{time} = \frac{95 \text{ km}}{\text{hr}} \cdot \frac{3}{60} \text{ hr} = 4.75 \text{ km}
\]

6) (10 pts) A person exerts an upward force of 40 N to hold a bag of groceries. Describe the “reaction force” by giving its a) magnitude and direction, b) on what body it is exerted, and c) by what body it is exerted.

- \( \text{reaction force} = 40 \text{ N down} \)
- \( \text{on hands/arms of person} \)
- \( \text{by bag of groceries} \)

7) (10 pts) A block is given a push so that it slides up a ramp. After the block reaches its highest point, it slides back down. Why is the magnitude of its acceleration less on the descent than on the ascent? Explain in words, and also with a labeled free-body diagram.

**Ascent**

- \( F_N \)
- \( F_m g \)
- Both friction and gravity act to slow down the block

**Descent**

- \( F_N \)
- \( F_m g \)
- On descent, gravity increases the block's speed, but friction works against gravity, slowing block down.

**THUS:** greater net force during ascent than during descent!
8) (10 pts) A 3.0-kg block is suspended by a massless cord from another 3.0-kg block, which in turn is hung by a second massless cord.

   a) If the blocks are at rest, find the tension in each cord.

   \[ \text{lower block: } ma = 0 = T_2 - mg \quad \text{(take up as pos.)} \]
   \[ \text{upper block: } ma = 0 = T_1 - mg - T_2 \]

   \[ T_2 = mg = (3 \text{ kg}) (9.8 \text{ m/s}^2) = 29 \text{ N} \]
   \[ T_1 = mg + T_2 = 58 \text{ N} \]

   b) If the two blocks are pulled upward with an acceleration of 2.0 m/s\(^2\), find the tension in each cord.

   \[ \text{Now } a = 2m/\text{s}^2 \text{ up for both blocks!} \]
   \[ T_2 = ma + mg = m \left( 2.0 \text{ m/s}^2 + 9.8 \text{ m/s}^2 \right) = 35.4 \text{ N} \]
   \[ T_1 = ma + mg + T_2 = 70.8 \text{ N} \]

9) (10 pts) A 200-kg elevator car is designed to reach a maximum upward acceleration of 1.2 m/s\(^2\). The maximum allowed tension in the elevator cable is 10,000 N.

   a) Assuming the average elevator occupant has a mass of 70 kg, how many people should be permitted to ride the elevator at one time?

   \[ ma = T - mg \quad \text{or} \quad m(a + g) = T \quad \text{or} \quad m = \frac{T}{(1.2 + 9.8) \text{ m/s}^2} \]
   \[ m = 909 \text{ kg} = \text{total allowed mass} \]
   \[ - \text{elevator mass} \]
   \[ = \text{people mass} \]
   \[ \frac{-200}{909 \text{ kg}} = \frac{10}{11} = \text{too many!} \]

   b) Suppose instead that the elevator is accelerating downward at 1.2 m/s\(^2\). Under this condition, how many average people can safely ride the elevator?

   \[ m(a + g) = T \]
   \[ m = \frac{10,000}{(9.8 - 1.2) \text{ m/s}^2} = \frac{1163 \text{ kg total mass}}{963 \text{ kg for people}} \]
   \[ - \text{200 kg elevator} \]
   \[ = 13.75 \text{ average people} \]
   \[ \text{Max load: 13 people, but too many!} \]
10) (10 pts) A flat puck of M = 100 grams is rotated in a circle on a frictionless tabletop, and is held in this orbit by a light cord which is connected to a dangling mass of m = 10 kg through the central hole shown in the figure. The radius of the circle is 10 cm. Find the speed of the puck.

![Diagram of puck and cord with equations]

Centripetal force provided by tension T in cord, \( T = mg \)

So \( F_c = \frac{Mv^2}{R} = mg \) or \( v = \sqrt{\frac{Rmg}{M}} \)

\[
= \sqrt{\frac{(0.10 \text{ m})(10 \text{ kg})(9.8 \text{ m/s}^2)}{0.1 \text{ kg}}} = 9.9 \text{ m/s} \approx 10 \text{ m/s}
\]

11) (10 pts) Four 7.5 kg spheres are located at the corners of a square with side length of 0.60 m. A fifth sphere of mass 10.0 kg is found at the center of the square. Find the magnitude and direction of the gravitational force acting on the 10.0 kg sphere.

![Diagram of spheres with equations and text]

Four equal forces \( F \) act on central mass \( M \) but cancel each other by symmetry!

\((\text{Ignoring Earth's gravity})\)

Zero net force!

12) (10 pts) Derive a formula for the mass of a planet in terms of its radius \( r \), the acceleration due to gravity at its surface \( g_p \), and the gravitational constant \( G \).

\[
M = \text{mass of planet}
\]
\[
m = \text{mass of object on planet's surface}
\]

\[
F = mg_p = \frac{GMm}{r^2}
\]

So \( M = \frac{r^2 g_p}{G} \)
Useful Equations

For $ax^2 + bx + c = 0$, solutions to the quadratic are $x = (1/2a) [-b \pm (b^2 - 4ac)^{1/2}]$

$v^2 = v_0^2 + 2a(x-x_0)$ \quad $x(t) = x_0 + v_0t + \frac{1}{2}at^2$ \quad $v = v_0 + at$ \quad $F_{\text{net}} = ma$

$G = 6.67 \cdot 10^{-11}$ N$m^2$/kg$^2$ \quad $a_c = v^2/r$ \quad $F = Gm_1m_2/r^2$